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NOZZLE

N. V. Kokushkin

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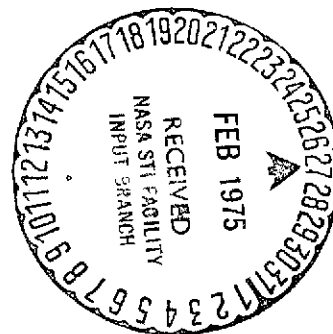
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16. Abstract A method of algebraic equations is derived for determining the reflection characteristics of acoustic waves in the subcritical section of a Laval nozzle during combustion. The coefficient matrix of these equations is used as the basic characteristic of the acoustic property of the nozzle. The amplitudes of reflected acoustic waves are also determined from the amplitudes of incident acoustic waves. A linear boundary value problem is solved in the process.			
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N. V. Kokushkin

NOTATION

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- ρ_N, θ_N, r_N - Spherical coordinates referred to the N-th coordinate system;
- x_N, r_N, r_N - Cylindrical coordinates referred to the N-th coordinate system;
- Φ - Acoustic potential;
- \mathbf{v}' = grad Φ' - Vector of acoustic velocity;
- \mathbf{v}_0 - Vector of stationary velocity;
- c - Speed of sound;
- t - Time;
- ω - Circular frequency;
- m - Mode of wave components;
- M - Mach number;
- β_j - Sequence of roots of equation $\frac{d}{dr} J_m(\beta_j r) = 0$ when $r = r_c$ (r_c - radius of cylindrical section of chamber);
- n_j - Sequence of roots of equation $\frac{d}{d\theta} P_n(\cos \theta) = 0$ when $\theta = \theta_0$
- $(\theta_0$ - Angle of opening of nozzle conical section).

* Numbers in margin indicate pagination in original foreign text.

In a study of the high frequency ("acoustic") instability of the operational process in combustion chambers of different types, the necessity arises of determining the reflection of acoustic waves from the subcritical section of supersonic nozzles.

Several works have investigated this phenomenon [1-7]. An attempt is made in [1-3] to determine the acoustic impedance of the nozzle by solving an ordinary linear differential equation of the second order with variable coefficients, describing the propagation of plane acoustic waves in a flow, whose velocity changes along the coordinate (one-dimensional and linear formulation of the problem).

Thus, the authors have investigated the propagation of plane [1-3] and three-dimensional [4, 5] acoustic waves in a cylindrical tube under the condition of a given change in the axial stationary/119 velocity component of the flow.

With this formulation of the problem, the effect of reflection is determined only by an inhomogeneity of the stationary flow. It is thus impossible to take into consideration the reflection directly from the walls of the nozzle contracting section.

This article proposes a method for solving the problem of the reflection of acoustic waves from the nozzle subcritical section. A real nozzle is replaced by a certain model of a nozzle made up of conical sections of "zones" (Figure 1). Each of these zones coincides with a certain section of the flow in a conical channel having the same angle of contraction.

It is assumed that the direction of the velocity of the stationary flow is opposite to the direction of the radius vector ρ_N of spherical coordinates. It is assumed that the magnitude of

the stationary flow velocity is given; however, its values must not depend on the angular coordinate θ_N .

Under these conditions, it is possible to obtain particular solutions of the equation of the acoustic potential in the normal form, i.e., in the form of the product of the functions, each of which depends only on one of the independent variables of the curvilinear coordinates and time.

The general solution for the conical channel is obtained by summing the particular solutions of such a type [8]. The acoustic field in the model of the subcritical nozzle section (Figure 1) is composed of sections of the solutions for the corresponding conical channels. A system of algebraic equations, which connects the amplitudes of the waves incident on the nozzle with the reflected waves, is obtained from the condition that the values of these solutions and their derivatives coincide at the boundaries of the zones. The matrix of the coefficients of this system is the desired characteristic of the reflective properties of the nozzle. It is a generalization of the amplitude reflection coefficient known from acoustics [8]. This method may be developed for nozzles with a smooth change in the contour of the subcritical section.

The solutions of the equation of the acoustic potential

$$\nabla^2 \varphi' - \frac{1}{c^2} \left\{ \frac{\partial^2 \varphi'}{\partial t^2} + \frac{\partial}{\partial t} [2(\mathbf{v}_0 \mathbf{v}')] + [\mathbf{v}_0 \text{grad}(\mathbf{v}_0 \mathbf{v}')] + \left(\mathbf{v}' \text{grad} \frac{|\mathbf{v}_0|^2}{2} \right) \right\} = 0$$

in the case when the dependence on time may be taken into account by the factor $e^{i\omega t}$ have the form

$$\varphi' = \sum_m (C_{1m}^{(N)} \cos m\eta + C_{2m}^{(N)} \sin m\eta) \times \sum_{j=0}^{\infty} [C_{1mj}^{(N)} X_{1mj}(x) + C_{2mj}^{(N)} X_{2mj}(x)] J_m(\beta_j r) \quad (1)$$

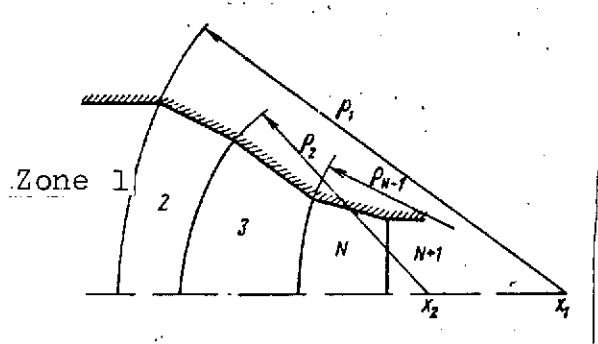


Figure 1. Diagram of subcritical nozzle sections.

in cylindrical systems of coordinates (when $N = 1$ and $N = N^* + 1$) and

$$\varphi'_S = \sum_m (C_{1m}^{(N)} \cos m\eta + C_{2m}^{(N)} \sin m\eta) \times \left| \sum_{j=0}^{\infty} [C_{1pmj}^{(N)} R_{1mj}(\rho) + C_{2pmj}^{(N)} R_{2mj}(\rho)] P_n^m(0_N) \right| \quad (2)$$

in spherical coordinate systems ($N=2, \dots, N^*$).

Here $X_{1mj}(x)$ and $X_{2mj}(x)$ reflect the dependence of the acoustic potential on the x coordinate for incident and reflected waves in a cylindrical channel.

Thus $R_{1mj}(\rho)$ are regular, when $M = 1$ and $R_{2mj}(\rho)$ are irregular solutions of Equation (8), given below.

The requirement that the acoustic potential and its derivative be equal, which are described in coordinate systems of conical flows as applied to two adjacent zones (N -th and $N+1$ -th), gives the following relationship for each individual mode of oscillations (m) at the "input" boundary of the $(N+1)$ th zone

$$\begin{aligned} \varphi'_N &= \varphi'_{N+1}, \\ \partial \varphi'_N / \partial \rho_{N+1} &= \partial \varphi'_{N+1} / \partial \rho_{N+1}, \quad N = 1, 2, \dots, N^*. \end{aligned} \quad (3)$$

It must be noted that no conditions are imposed on the values of the stationary flow velocity vector close to the surface of the solutions which are combined, and the velocity on both sides of this surface may differ in terms of magnitude and direction. As the calculations and special determinations have shown, small inaccuracies in the picture of the stationary flow have no great influence upon the parameters of the acoustic field.

When the solutions are "combined", the relationship is used between the coordinates $(\rho_N, \theta_N, \eta_N)$ and $(\rho_{N+1}, \theta_{N+1}, \eta_{N+1})$ in two adjacent zones, obtained from purely geometric considerations

$$\left. \begin{aligned} \rho_N &= \rho_{N+1} \frac{\sin(\pi - \theta_{N+1})}{\sin \left\{ \frac{\theta_{N+1}}{2} - \arctg \left[\frac{\Delta x_{N,N+1} - \rho_{N+1}}{\Delta x_{N,N+1} + \rho_{N+1}} \operatorname{ctg} \left(\frac{\pi - \theta_{N+1}}{2} \right) \right] \right\}}, \\ \theta_N &= \theta_{N+1} \cdot \frac{1}{2} - \arctg \left[\frac{\Delta x_{N,N+1} - \rho_{N+1}}{\Delta x_{N,N+1} + \rho_{N+1}} \operatorname{ctg} \left(\frac{\pi - \theta_{N+1}}{2} \right) \right], \quad \eta_N = \eta_{N+1} \end{aligned} \right\}$$

in the case of "combining" of the conical region with the conical and

$$\left. \begin{aligned} x^{(k)} &= x_0 + \rho_1 \cos \theta_1, \\ r^{(k)} &= \rho_1 \sin \theta_1 \end{aligned} \right\}$$

in the case of "combining" of the cylindrical region of the combustion chamber with the conical region II (Figure 1).

Multiplying Equation (3) by $P_{nj}(\theta)$ and integrating over the corresponding interval of the change in the angle $0 \leq \theta \leq \theta_{(N+1)}$ and using the conditions of orthogonality of the systems of [attached] spherical Legendre functions, we obtain a system of algebraic equations.

It must be assumed that the values of the coefficients $C_{1nj}^{(N)}$ in the solution (1) when $N = 1$ in a combustion chamber, which characterize the amplitudes of the waves incident upon the nozzle,

are given, i.e., the system of algebraic equations obtained is an inhomogeneous system, and the terms containing the factors $C_{1xm}^{(N)}$ form the right hand sides of the equations.

The absence of a singularity for the solution close to the critical nozzle section is expressed by the condition

$$C_{2xm}^{(N)} = 0 \text{ at } N = N^* + 1.$$

All the remaining values of C must be defined, including the values of $C_{2xm}^{(0)}$, characterizing the waves reflected from the nozzle in the cylindrical section of the combustion chamber. /121

Since the field of the acoustic potential in the general case is expressed by infinite series (1), (2), the system of algebraic equations obtained will be an infinite system. Therefore, in actual calculations, we must investigate a certain reduced system of equations.

The matrix of this system does not depend on the form of the waves incident on the nozzle and is the acoustic characteristic of the nozzle, which may be used when solving the problems of oscillations and the stability of the operational process in the combustion chambers.

To obtain complete agreement between the geometric model of a real nozzle and a smooth change of the contour, in the relationships obtained it is necessary to pass to the limit when $N \rightarrow \infty$, i.e., increasing the number of zones into which the entire flow field is divided in the subsonic nozzle section.

It must also be required that the length of the zones in the direction of the radius vector of the spherical coordinates ρ_N uniformly strive to zero in the $0 < x < L$ interval and that an unequivocal

agreement hold between the number of the zone $N \leq N^*$ and a certain point $x^{(N)}$ of the axial coordinate, for example, $N = N^* (1 - x^{(N)}/L)$.

Under these conditions, the sequences of the corresponding values — for example, $R_{2mj}^{(1)}, R_{2mj}^{(2)}, \dots, R_{2mj}^{(N)}, \dots, R_{2mj}^{(N^*+1)}, C_{1\rho mj}^{(1)}, C_{1\rho mj}^{(2)}, \dots, C_{1\rho mj}^{(N^*+1)}$ and others — will more correctly represent a certain function of the N - number of the zone and at the limit will change into continuous functional dependences

$$\left. \begin{aligned} R_{2mj}^{(N)} &\rightarrow R_{2j}(x^{(N)}), \\ C_{1\rho mj}^{(N)} &\rightarrow C_{1j}(x^{(N)}) \end{aligned} \right| \text{ etc.}$$

In accordance with this, the series of algebraic equations with identical values of the index j , indicating the number of the row in the blocks of the matrix, are transformed into the relationship between the functions of the argument $x^{(N)}$:

$$\begin{aligned} &\sum_{\nu} C_{1\nu}(x^{(N)}) A_{\nu j}(x^{(N)}) - C_{1j}(x^{(N)} + \Delta x^{(N)}) R_{1j}(x^{(N)}) + \\ &+ \sum_{\nu} C_{2\nu}(x^{(N)}) B_{\nu j}(x^{(N)}) - C_{2j}(x^{(N)} + \Delta x^{(N)}) R_{2j}(x^{(N)}) = 0, \\ &\sum_{\nu} C_{1\nu}(x^{(N)}) A'_{\nu j}(x^{(N)}) - C_{1j}(x^{(N)} + \Delta x^{(N)}) R'_{1j}(x^{(N)}) + \\ &+ \sum_{\nu} C_{2\nu}(x^{(N)}) B'_{\nu j}(x^{(N)}) - C_{2j}(x^{(N)} + \Delta x^{(N)}) R'_{2j}(x^{(N)}) = 0, \quad j, \nu = 0, 1, 2, 3, \dots, \nu, \end{aligned} \quad (5)$$

where we set

$$\begin{aligned} \Delta x^{(N)} &= x^{(N+1)} - x^{(N)}, \\ \cos \theta_{0(N)} &= \int_0^1 R_{1mj}(\rho, \theta_N) P_{nj}^m(\rho_N, \theta_N) P_{n\nu}^m(\theta_N) d(\cos \theta_N); \end{aligned} \quad (6)$$

$$\begin{aligned} \cos \theta_{0(N)} &= \int_0^1 R_{2mj}(\rho_N, \theta_N) P_{nj}^m(\rho_N, \theta_N) P_{n\nu}^m(\theta_N) d(\cos \theta_N) \end{aligned}$$

- are the coefficients of the expansion in series of the functions $(R_{1m}P_{nj}^m)$ and $(R_{2m}P_{nj}^m)$ in powers of P_{nv}^m .

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Substituting the expressions

$$C_{1j}(x + \Delta x) \cong C_{1j}(x) + \frac{dC_{1j}}{dx} \Delta x,$$

$$C_{2j}(x + \Delta x) \cong C_{2j}(x) + \frac{dC_{2j}}{dx} \Delta x$$

in (5) and setting

$$A_{1vj} = \lim_{\Delta x \rightarrow 0} \frac{A_{vj}R_{2j}' - A_{vj}'R_{2j}}{(R_{1j}R_{2j}' - R_{1j}'R_{2j}) \Delta x},$$

$$A_{2vj} = \lim_{\Delta x \rightarrow 0} \frac{A_{vj}R_{1j}' - A_{vj}'R_{1j}}{(R_{2j}R_{1j}' - R_{2j}'R_{1j}) \Delta x},$$

$$B_{1vj} = \lim_{\Delta x \rightarrow 0} \frac{B_{vj}R_{2j}' - B_{vj}'R_{2j}}{(R_{1j}R_{2j}' - R_{1j}'R_{2j}) \Delta x},$$

$$B_{2vj} = \lim_{\Delta x \rightarrow 0} \frac{B_{vj}R_{1j}' - B_{vj}'R_{1j}}{(R_{2j}R_{1j}' - R_{2j}'R_{1j}) \Delta x}$$

where $v \neq j$,

$$A_{1vv} = \lim_{\Delta x \rightarrow 0} \frac{(A_{vv} - R_{1v})R_{2v}' - (A_{vv}' - R_{1v}')R_{2v}}{(R_{1v}R_{2v}' - R_{1v}'R_{2v}) \Delta x},$$

$$A_{2vv} = \lim_{\Delta x \rightarrow 0} \frac{(A_{vv} - R_{1v})R_{1v}' - (A_{vv}' - R_{1v}')R_{1v}}{(R_{2v}R_{1v}' - R_{2v}'R_{1v}) \Delta x},$$

$$B_{1vv} = \lim_{\Delta x \rightarrow 0} \frac{(B_{vv} - R_{2v})R_{2v}' - (B_{vv}' - R_{2v}')R_{2v}}{(R_{1v}R_{2v}' - R_{1v}'R_{2v}) \Delta x},$$

$$B_{2vv} = \lim_{\Delta x \rightarrow 0} \frac{(B_{vv} - R_{2v})R_{1v}' - (B_{vv}' - R_{2v}')R_{1v}}{(R_{2v}R_{1v}' - R_{2v}'R_{1v}) \Delta x}$$

where $v=j$, relationships (5) may be transformed into the following differential equations

$$\frac{dC_{1j}}{dx} = \sum_{v=0}^{\hat{v}} (C_{1v}A_{1vj} + C_{2v}B_{1vj}),$$

$$\frac{dC_{2j}}{dx} = \sum_{v=0}^{\hat{v}} (C_{1v}A_{2vj} + C_{2v}B_{2vj}),$$

$$j = 0, 1, 2, \dots, \hat{v}.$$

The values of the amplitudes of the waves incident upon the nozzle at the input section $x = L$ may be used as the boundary conditions, and the fact must be taken into account that close to the critical section, there will be no components expressed by the functions $R_{2mj}^{(N^{*+1})}/$, having a singularity in the critical section at $M = 1$, i.e., all $C_{2mj}^{(N^{*+1})} = 0$

$$\left. \begin{aligned} C_{1j}(L) &= a_j, \\ C_{2j}(0) &= 0, \\ j &= 0, 1, 2, \dots, \hat{v}. \end{aligned} \right\}$$

Thus, the problem of the reflection of acoustic waves from the subcritical section of a Laval nozzle is reduced to the solution of the linear boundary value problem for an infinite system of differential equations. If we confine ourselves to an examination of a finite number of differential equations $(v, j \leq \hat{v})$, the linear homogeneous boundary value problem with these boundary conditions may lead to the solution $(n+1)=2(\hat{v}+1)+1$ of the Cauchy problem [9] (n - order of magnitude of the system).

A model consisting of a cylindrical section of the combustion chamber combined directly with a conical convergent channel is the simplest variation of the geometric model of a subsonic section of the nozzle. If it is assumed that the region of transition through the speed of sound is close, in terms of its geometric form, to a spherical segment which "closes" the output from the convergent channel, then there must be no solutions having a singularity at $M = 1$ in the representation of the acoustic field in spherical coordinates in the conical section of the nozzle:

$$C_{2mj}^{(2)} = 0.$$

An examination of this problem is interesting, due to the fact that it makes it possible to determine the effect of reflection from a single "bend" of the reflecting rigid shell of the chamber — the nozzle, and in this sense it characterizes one of

the elements comprising the geometric model of the nozzle of a more complex form. In addition, the results obtained in the solution of this problem satisfactorily coincide with the known experimental data, and therefore they are of practical importance.

In this case the system of equations has the form

$$\left. \begin{aligned} \sum_{v=0}^{\hat{v}} C_{2xmv}^{(I)} B_{mvj}^{(I)} - C_{1pmv}^{(II)} R_{1mj}^{(II)} &= \sum_{v=0}^{\hat{v}} C_{1xmv}^{(I)} A_{mvj}^{(I)}, \\ \sum_{v=0}^{\hat{v}} C_{2xmv}^{(I)} B_{mvj}^{(II')} - C_{1pmv}^{(II)} R_{1mj}^{(II')} &= \sum_{v=0}^{\hat{v}} C_{1xmv}^{(I)} A_{mvj}^{(II')}, \\ j &= 0, 1, 2, \dots, \hat{v}. \end{aligned} \right| \quad (7)$$

In the calculations, it is advantageous to study the effect of reflection for each of the wave components of the [incident] wave individually. This means that only one of the values $C_{1xmv}^{(I)}$ differs from zero. If the given component has the index $v=v'$, we may set $C_{1xmv}^{(I)} = 1, C_{1xmv}^{(I)} = 0$ for $v \neq v'$.

The coefficients $A_{mvj}^{(I)}$ and $A_{mvj}^{(II')}$ in the right side of the system (7), are calculated according to the formulas (6). The values of $B_{mvj}^{(I)}$ and $B_{mvj}^{(II')}$ are also calculated according to the formulas (6). The functions $R_{1mj}(x)$ and $R_{2mj}(x)$ in them have the following form in this case

$$\left. \begin{aligned} X_{1mj} &= \exp i \frac{kM + \sqrt{k^2 - \beta_j^2 (1 - M^2)}}{1 - M^2} x, \\ X_{2mj} &= \exp i \frac{kM - \sqrt{k^2 - \beta_j^2 (1 - M^2)}}{1 - M^2} x. \end{aligned} \right|$$

This is the solution of the equation

$$X''(1 - M^2) - X' \left(2ikM + 2M \frac{\partial M}{\partial x} \right) + X(k^2 - \beta_j^2) = 0.$$

at $M = \text{const}$, which reflects the waves which are incident (X_{1mj}) and reflected (X_{2mj}) in the cylindrical section of the combustion chamber. The value of k reflects the dimensionless frequency $k = \omega \rho_{\text{cr}}/c$, where ρ_{cr} — spherical radius of the critical section. The Legendre functions P_n^m may be replaced by the Bessel functions.

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The functions $R_{lmj}^{(1)}$ and their derivatives with respect to ρ are solutions of the equation

$$R''(1 - M^2) + R' \left(\frac{2}{\rho} + 2ik[M] - 2M \frac{dM}{d\rho} \right) + \left[k^2 - \frac{n_j(n_j + 1)}{\rho^2} \right] R = 0, \quad (8)$$

which are regular when $\rho = \rho_{\text{cr}}$. The dependence of the Mach number on the spherical coordinate ρ is assumed to be given.

In the calculations of this study, it is given in the form

$$M = m_2 \left(\frac{\rho_{\text{cr}}}{\rho} \right)^2 + m_6 \left(\frac{\rho_{\text{cr}}}{\rho} \right)^6 + m_{10} \left(\frac{\rho_{\text{cr}}}{\rho} \right)^{10}$$

i.e., in the form of the first three terms of the expansion in series of the gas dynamic function $q(M)$ [10]. The values of the coefficients m_i are functions of the adiabatic index. However, their values must be "corrected" with respect to the condition $\sum_i m_i = 1$ since

$$M(\rho_{\text{cr}}) = 1$$

All of the calculational results given below were obtained on computers of the BESM-3M and BESM-4 types. The programs were compiled using the α -translator, with standard programs and procedures. The duration of a calculation of one variation, i.e., the calculation of the acoustic field of the nozzle of a specific geometric form (ρ_1, θ_0) , when a certain wave $(k, C_{1xmv}^{(1)})$

is incident on the nozzle, is 25-30 minutes.

The maximum order of the wave components examined was five when $(v+1=5)$ which leads to the solution of the system of algebraic equations with real coefficients of the 20th order (the SP-0143 was used).

θ_0°	k	ρ_1	Amplitude $ C_{21} $				
			$n^* = 1$	$n^* = 2$	$n^* = 3$	$n^* = 4$	$n^* = 5$
15	1.0	1.5*	0,525268	0,525612	0,525651	0,525662	0,525667
15	3.0	1.5	0,2443	0,24305	0,24236	0,24199	0,24177
15	6.0	1.5	0,05344	0,064826	0,06646	0,06758	0,067404
30	1.0	1.05	0,1055	0,0974	0,1011	0,1003	0,1006
30	1.0	2.0	0,6491	0,65354	0,653382	0,653414	0,653409
30	1.0	3.0	0,63055	0,62518	0,62335	0,62241	0,62189
30	6.0	1.5	0,05094	0,04321	0,048006	0,04928	0,04949
30	10.0	1.5	0,0366	0,0373	0,0386	0,378	0,0405
45	0.1	1.5	0,56477	0,56478	0,56480	0,56483	0,56481
45	1.0	1.5	0,5324	0,5408	0,5366	0,5377	0,5374
45	3.0	1.5	0,3070	0,4295	0,3719	0,3898	0,3844
45	5.0	1.5	0,0616	0,1142	0,1421	0,1315	0,1332
60	1.0	1.5	0,546	0,560	0,541	0,552	0,547
60	3.0	1.5	0,422	0,375	0,354	0,360	0,362
75	0.1	1.5	0,5652	0,5654	0,5648	0,5653	0,5649
75	0.5	1.5	0,567	0,582	0,553	0,584	0,553
90	0.01	1.5	0,56511	0,56514	0,56505	--	--
90	0.001	1.5	0,565104	0,565107	0,565104	--	--

* Translator's note: Commas in numbers represent decimal points.

During the calculations, a comparison was made of the results obtained for different values of $n^* = v+1$. Some of these values are given in the table. The values are given of the amplitude of the first wave component of the reflected wave $|C_{21}|$ (i.e., a "plane" wave) in the case of the incidence on the nozzle of the first wave component $v^*=0$ (i.e., the "plane" wave) for different frequencies k and nozzles of differing geometric form (ρ_1, θ_0) . The data given point to a great ("rapid") convergence of the C_{21} computational results with an increase in the number n^* of the investigated terms of the series of expansions (1), (2) at small angles of the conical subsonic section of the nozzles (the angles $\theta_0 = 15^\circ, 30^\circ, 45^\circ$).

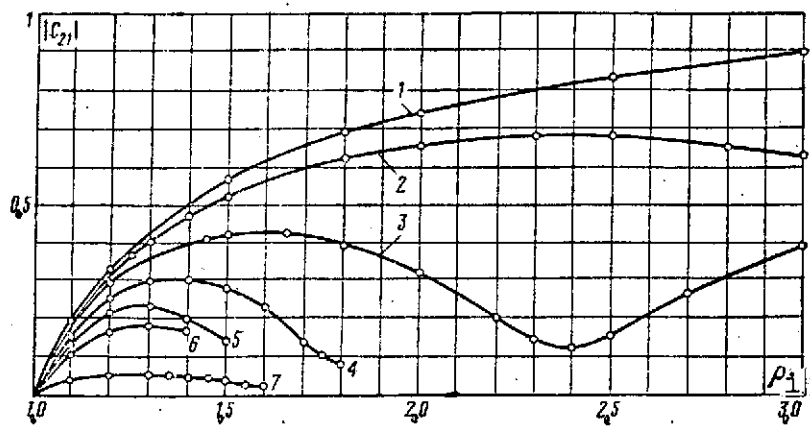


Figure 2. Dependence of reflection coefficient for potential amplitude upon length of subcritical nozzle section at $\theta_0 = 30^\circ$ for different oscillation frequencies k . Plane waves ($m=0$, $v^*=0$): 1- $k=0.1$; 2- 1.0 ; 3- 2.0 ; 4- 3.0 ; 5- 4.0 ; 6- 5.0 ; 7- 10.0

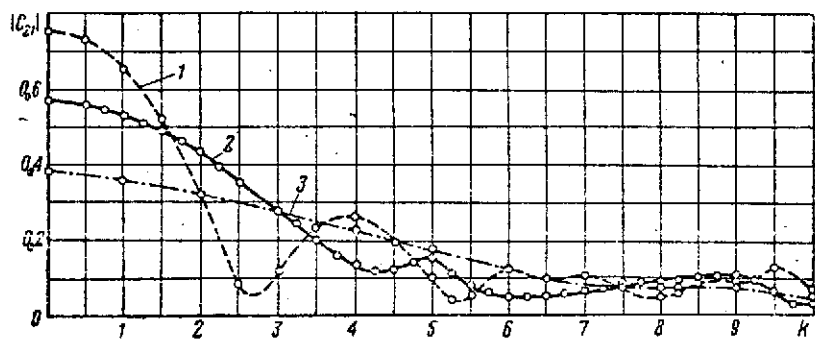


Figure 3. Dependence of the C_{21} coefficient modulus of the potential amplitude of the first wave component of the reflected wave on the frequency k for nozzles with differing lengths of the subcritical section ($m = 0$; $v^* = 0$; $\theta_0 = 30^\circ$):

1- $\rho_1 = 2.0$; 2- 1.5 ; 3- 1.25

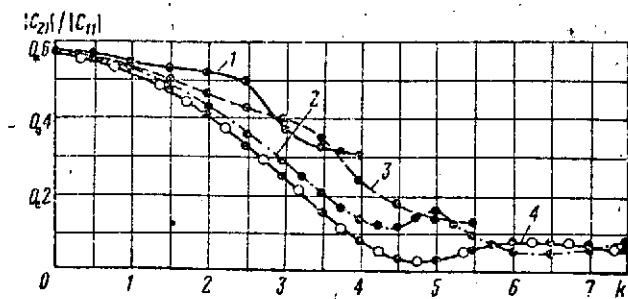


Figure 4. Dependence of reflection coefficient modulus for potential amplitude upon the frequency k for nozzles with differing angles of the subcritical section cone ($m = 0$; $\rho_1 = 1.5$; $v^* = 0$):

1- $\theta_0 = 60^\circ$; 2- 30° ; 3- 45° ; 4- 15° .

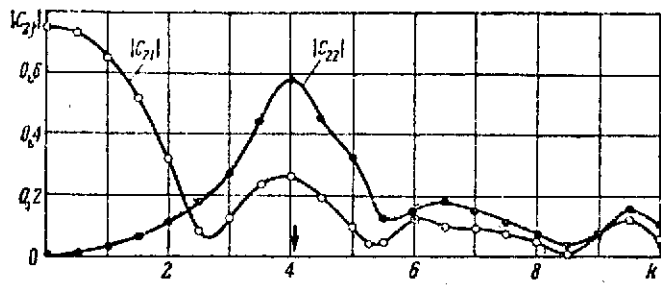


Figure 5. Dependence on frequency of the coefficients C_{21} and C_{22} of the amplitudes of the first two wave components of a reflected wave when a plane wave is incident on the nozzle ($C_{11} = 1$; $m = 0$; $v^* = 0$; $\theta_0 = 30^\circ$; $\rho_1 = 2.0$); the arrow indicates the critical tube frequency.]

With an increase in the angle and an increase in the frequency k , the convergence deteriorates. Thus, at $\theta_0 = 75^\circ, 90^\circ$, the calculations are only possible (at $n = v+1=5$) for very low frequencies ($k = 0.01; 0.001$). To increase the accuracy of the results under these conditions, it is necessary to examine a larger number of terms in the series. /126

Figure 2 shows the dependence of the wave component of a reflected wave on the length of the subsonic nozzle section at different frequencies and angle of the cone $\theta_0 = 30^\circ$. A decrease in the reflection is characteristic with the decrease in the length of the subsonic section, i.e., with an increase in the "opening" of the combustion chamber and an increase in the Mach number at the input to the nozzle

$$|C_{21}| \rightarrow 0 \text{ at } \rho_1 \rightarrow 1, \hat{\rho}_{cr} = 1$$

The reflection of the "longitudinal" wave also decreases with an increase in the frequency k .

Figure 3 shows the frequency characteristics of these wave components for nozzles of differing lengths, and Figure 4 — for

nozzles with differing input angles. It can be seen from these figures that at $k \rightarrow 0$ the reflection coefficient does not depend on the geometric form of the subsonic section (angle θ_0) but depends on p_1 , i.e., on the Mach number at the nozzle inlet. From the physical point of view this can be explained: with a decrease in the frequency and an increase in the wavelength, any of the nozzles, except for the dependence on its shape, may be regarded as a "sudden" decrease in the channel section. With an increase in the frequency, the reflection coefficient decreases. The existing characteristic maxima correspond, as a rule, to the critical values of the frequency

$$k = \beta_1(1 - M^2)^{1/2}.$$

The comparison, shown in Figure 5, of the amplitudes of the first ("plane", "longitudinal") wave component of the reflected wave and the second ("radial") wave component of the reflected wave, in the case of incidence of "plane" and "longitudinal" waves, shows that as the frequency approaches its critical value the "radial" component in the reflected wave greatly exceeds the "longitudinal and plane" component. This shows that at such frequencies, it is not permissible to examine the problems of stability of the operational process in the combustion chamber in the one-dimensional formulation.

There are limited possibilities for comparing the results of the calculations with experimental data. Out of the experimental investigations on this problem which are known in the literature, only the experimental conditions used in [2,7] can be used to make such a comparison. The study [2] measured the damping decrement of the oscillations α , and the acoustic quality Q under the conditions of a model combustion chamber (at one frequency $f_0 \approx 548$ Hz) in the case of cold air. The ratio of the

areas of the critical nozzle-disk section and the pass-through section of the chamber $j = F_{cr}/F_{ch}$ $0 \leq j \leq 0.4$ were varied. The results are given in the form of the dependence of α on j . Utilizing the relationship

$$\alpha = \frac{E}{2E} = \frac{E \cdot 2 \left(1 - \frac{|C_{21}|}{|C_{11}|}\right) \frac{c}{a} + E \frac{V_0}{a}}{2E} = \left(1 - \frac{|C_{21}|}{|C_{11}|}\right) \frac{c}{a} + \frac{1}{2} M \frac{c}{a},$$

in which the second term expresses the convective output of acoustic energy, and a —the chamber length, we may express the amplitude reflection coefficient by the value of α measured in [2] and the Mach number, which is unequivocally connected with the parameter j , and $j = \rho_{cr}^2 / \rho_i^2$. Since a very short "nozzle"-disk was used in the experiments, a comparison must only be made with the results of calculations at $k \rightarrow 0$. Figure 6 gives such a comparison.

The data of the experimental study given in [6] pertain to transverse oscillations ($m = 1$). On the graphs in this study, the change of the real and imaginary section of the acoustic conductivity is extremely contracted along the ordinate axis, which makes it impossible to perform an objective comparison of the experimental and theoretical calculations due to the difficulty of accurately computing the data from the graphs [6].

Similar data for plane waves ($m=0$) are given in [7] in a form which is more advantageous for processing: the scale along the ordinate axis is increased by one order of magnitude.

Figure 7, a and b gives a comparison of the values of the amplitude reflection coefficients for pressure fluctuations $|R| = \frac{|1-y|}{|1+y|}$ obtained from data in [7] with the values calculated according to the proposed method on a computer. Since the frequency of the oscillations in the experiments was small (much

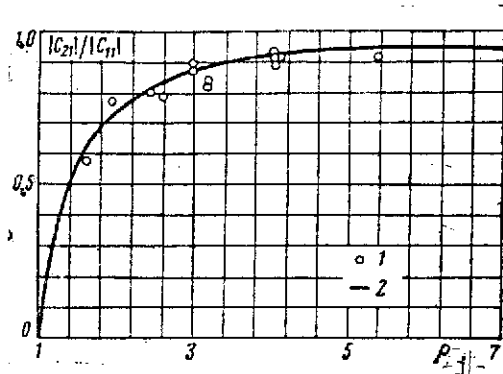


Figure 6. Comparison of experimental data from [2] with the calculated value when $k \rightarrow 0$; 1- experiments; 2- calculations.

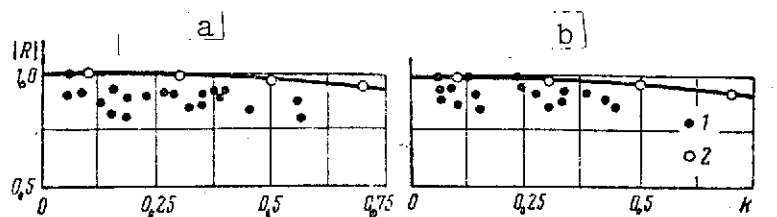


Figure 7. a. Comparison of experimental data [9] with calculations ($m = 0$; $M_1 = 0.08$): 1- experiment; 2- calculation.

b. Comparison of experimental data [9] with calculations ($m = 0$; $M_1 = 0.06$): 1- experiment; 2- calculation.

less than the critical frequency of the tube), the form of the nozzles could have no significant influence upon the reflection of acoustic waves. Therefore, the comparison was made with a certain conical nozzle having the same ratio of the input area to the critical section area. The relationship between the theoretically calculated value of $|C_{21}|/|C_{11}|$ and the reflection coefficient for the pressure amplitude $|R|$ is expressed in the case of stationary flow in the case of plane waves by the relationship

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$$|R| = \left| \frac{C_{21}}{C_{11}} \right| \cdot \frac{1 + |M|}{1 - |M|},$$

which may be readily obtained from (1), using the relationship between the acoustic potential and the acoustic pressure.

CONCLUSIONS

1. The method proposed makes it possible to solve the problem numerically in general form in a three-dimensional formulation with a small amount of machine time.
2. The characteristic of the acoustic properties of the nozzle, "the reflection matrix", may be used as a boundary condition when solving the problem of fluctuations in the combustion chamber in the case of composite multicomponent acoustic fields.
3. The basic relationships obtained for the reflection from the subcritical section of conical nozzles (dependence of reflection on the Mach number at the nozzle input, angle of contraction of the conical section, frequency characteristics) show that a) At low frequencies of oscillations, the form of the subcritical section of the nozzle has no great influence; b) The radial components play an important role in the formation of the acoustic field when the frequency approaches its critical value; in this frequency range, the stability problems cannot be examined in the one-dimensional formulation; c) There is a general tendency for a decrease in the components of the reflected wave with an increase in frequency; d) The results of the calculations are in satisfactory agreement with the known experimental data.

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